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THE AXIALLY SYMMETRIC TEMPERATURE PROBLEM FOR THE SPACE WITH A DISK-SHAPED CRACK

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One obtains an analytic solution for the axially quasistationary temperature problem of fracture mechanics for the space with a disk-shaped crack. The solution is constructed for an arbitrary time interval of the action of the load, as opposed to [1] where an asymptotics has been obtained only for small intervals of time.

The investigation of the growth mechanism of a crack under the conditions of a jump-like change of temperature has a great importance for the creation of the theoretical foundations of electric welding and for the evaluation of the rigidity of welded seams and joints.

1. We consider the axially symmetric state of stress of the unbounded space with a disk-shaped crack (circular in plane) (Fig. 1). We assume that at the part of the surface of the crack ($r < a$) a temperature T_0 arises at the initial instant, which remains constant in the sequel.

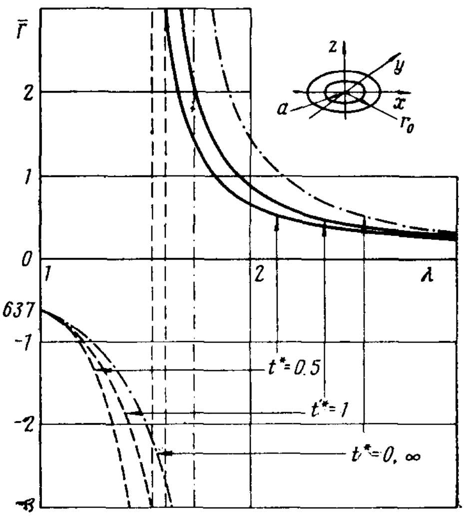

Fig. 1.

The problem of the determination of the axially symmetric temperature field reduces to solving the heat conduction equation [2]

$$\left(\frac{\partial^2}{\partial r^2} + \frac{1}{r}\frac{\partial}{\partial r} + \frac{\partial^2}{\partial z^2}\right) T - \frac{1}{\varkappa}\frac{\partial T}{\partial t} = 0 \qquad (1.1)$$

in the domain $z > 0$ with the following boundary conditions:

$$T(r, 0, t) = T_0(t), \quad r < a; \qquad \left.\frac{\partial T}{\partial z}\right|_{z=0} = 0, \quad r > a \qquad (1.2)$$

We rewrite (1.1), (1.2) by applying the Laplace-Carson transform with respect to time

$$\left(\frac{\partial^2}{\partial r^2} + \frac{1}{r}\frac{\partial}{\partial r} + \frac{\partial^2}{\partial z^2}\right) T^* - \frac{p}{\varkappa} T^* = 0 \qquad (1.3)$$

$$T^*(r, 0, p) = T_0^*(p), \quad r < a; \qquad \left.\frac{\partial T^*}{\partial z}\right|_{z=0} = 0, \quad r > a \qquad (1.4)$$

The solution of (1.3), which is bounded at infinity, can be represented in the form

$$T^*(r,z)=\int_0^\infty B(\alpha)\left(\alpha^2+\frac{p}{\varkappa}\right)^{-1/2}\exp\left(-z\sqrt{\alpha^2-\frac{p}{\varkappa}}\right)J_0(\alpha r)\,d\alpha \tag{1.5}$$

In order to satisfy conditions (1.4), we obtain for the determination of $B(\alpha)$ the dual integral equations

$$\int_0^\infty B(\alpha)\left(\alpha^2+\frac{p}{\varkappa}\right)^{-1/2}J_0(\alpha r)\,d\alpha=T_0^*(p),\qquad r\leqslant a \tag{1.6}$$

$$\int_0^\infty B(\alpha)J_0(\alpha r)\,d\alpha=0,\qquad r>a$$

We seek the solution of (1.6) in the form

$$B(\alpha)=\int_0^a\varphi(\xi)\sin\alpha\xi\,d\xi+Q_0\sin\alpha a \tag{1.7}$$

Here $\varphi(\xi)$ is the unknown function and Q_0 is some constant to be determined.

Taking into account the value of the integral [3]

$$\int_0^\infty J_0(\alpha r)\sin\alpha\xi\,d\alpha=\begin{cases}0, & \xi\leqslant r\\ (\xi^2-r^2)^{-1/2}, & \xi>r\end{cases} \tag{1.8}$$

we can see that (1.7) satisfies identically the second of the dual equations (1.6). Substituting (1.7) into the first of the equations (1.6) and interchanging the order of integration, we obtain

$$\int_0^a\varphi(\xi)\int_0^\infty J_0(\alpha r)\left(\alpha^2+\frac{p}{\varkappa}\right)^{-1/2}\sin\alpha\xi\,d\alpha\,d\xi+$$

$$+Q_0\int_0^\infty J_0(\alpha r)\left(\alpha^2+\frac{p}{\varkappa}\right)^{-1/2}\sin\alpha a\,d\alpha=T_0^*(p),\qquad r\leqslant a \tag{1.9}$$

We differentiate (1.9) with respect to r and we write this expression in the following form:

$$\int_0^\xi\frac{\xi\varphi(\xi)}{\sqrt{r^2-\xi^2}}\,d\xi=\int_0^a\varphi(\xi)\int_0^\infty\left[1-\alpha\left(\alpha^2+\frac{p}{\varkappa}\right)^{-1/2}\right]rJ_1(\alpha r)\sin\alpha\xi\,d\alpha\,d\xi+$$

$$+Q_0\int_0^a\left[1-\alpha\left(\alpha^2+\frac{p}{\varkappa}\right)^{-1/2}\right]J_1(\alpha r)\sin\alpha a\,d\alpha,\qquad r<a \tag{1.10}$$

For the derivation of the last equality we have made use of the known result [3]

$$\int_0^\infty J_1(\alpha r)\sin\alpha\xi\,d\alpha=\begin{cases}\xi r^{-1}(r^2-\xi^2)^{-1/2}, & \xi<r\\ 0 & \xi>r\end{cases}$$

Considering (1.10) as Schlömilch's equation [4], we find its solution in the form

$$\varphi(\xi)=\frac{2}{\pi}\int_0^a\varphi(\tau)\int_0^\infty\left[1-\alpha\left(\alpha^2+\frac{p}{\varkappa}\right)^{-1/2}\right]\sin\alpha\tau\sin\alpha\xi\,d\alpha\,d\tau+$$

$$+\frac{2}{\pi}Q_0\int_0^\infty\left[1-\alpha\left(\alpha^2+\frac{p}{\varkappa}\right)^{-1/2}\right]\sin\alpha a\sin\alpha\xi\,d\alpha \tag{1.11}$$

Thus, for the determination of $\varphi(\xi)$ we have obtained a Fredholm integral equation of the second kind with a symmetric kernel. In order to find an approximate solution of this equation we make use of the following approximation:

$$1-\alpha\left(\alpha^2+\frac{p}{\varkappa}\right)^{-1/2}\approx A_0\frac{p}{\varkappa}\left(\alpha^2+b^2\frac{p}{\varkappa}\right)^{-1} \quad (1.12)$$

This sort of ideas, related with the construction of approximate solutions as a result of the approximation of the kernel, has been developed by Koiter [5].

In Fig. 2 we have represented the function $y(\xi)=1-\xi(1+\xi^2)^{-1/2}$ $(\alpha=\xi\sqrt{p/\varkappa})$ (Curve 1) and its approximations for different values of A_0 and b^2. The curves 2—4 correspond to

$$2 - y(\xi)=\frac{0.5}{\xi^2+0.68}$$

$$3 - y(\xi)=\frac{0.5}{\xi^2+0.6}$$

$$4 - y(\xi)=\frac{0.5}{\xi^2+0.5}$$

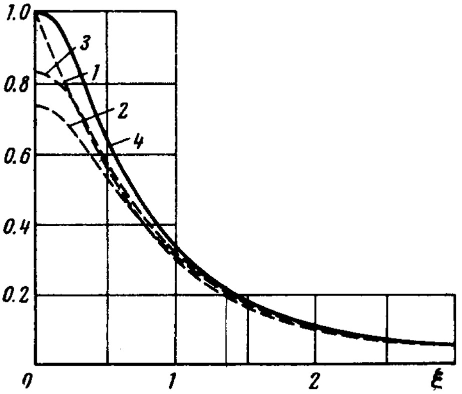

Fig. 2.

For the validity of this approxiamtion for $p\to\infty$ we will take in what follows $A_0=b^2=1/2$ (Curve 4). Making use of (1.12) we obtain an approximate expression for the kernel of Eq. (1.11)

$$\int_0^\infty[1-\alpha(\alpha^2+2q^2)^{-1/2}]\sin\alpha\tau\sin\alpha\xi\,d\alpha\approx q^2\int_0^\infty(\alpha^2+q^2)^{-1}\sin\alpha\tau\sin\alpha\xi\,d\alpha=$$

$$=\frac{1}{2}\pi q\,\mathrm{sh}(\omega q)\exp\left(-\frac{1}{\omega}\xi\tau q\right),\qquad \left(q=\sqrt{\frac{p}{2\varkappa}}\right) \quad (1.13)$$

$$\omega=\xi,\ \xi<\tau;\ \omega=\tau,\ \xi>\tau$$

Thus, Eq. (1.11) obtains the form

$$\varphi(\xi)=q\left[e^{-\xi q}\int_0^\xi\varphi(\tau)\,\mathrm{sh}(\tau q)\,d\tau+\mathrm{sh}(\xi q)\int_\xi^a\varphi(\tau)e^{-\tau q}d\tau+Q_0e^{-aq}\,\mathrm{sh}(\xi q)\right],\quad \xi<a \quad (1.14)$$

The solution of (1.14), satisfying the condition $\varphi(0)=0$ is obtained in the form

$$\varphi(\xi)=\frac{Q_0q^2}{1+aq}\xi \quad (1.15)$$

Substituting (1.15) into (1.7), we find

$$B(\alpha)\approx\frac{Q_0q^2}{1+aq}\int_0^a\xi\sin\alpha\xi d\xi+Q_0\sin\alpha a \quad (1.16)$$

We determine the distribution of the temperature in the plane by the formula (1.5) and by making use of the expression (1.16)

$$T^*(r,0)=\int_0^\infty B(\alpha)(\alpha^2+2q^2)^{-1/2}J_0(\alpha r)\,d\alpha\approx\frac{Q_0q^2}{1+\alpha q}\int_0^a\xi\int_0^\infty(\alpha^2+2q^2)^{-1/2}J_0(\alpha r)\times$$

$$\times \sin\alpha\xi\, d\alpha d\xi + Q_0 \int_0^\infty (\alpha^2 + 2q^2)^{-1/2} J_0(\alpha r) \sin \alpha a\, d\alpha =$$

$$= \frac{Q_0 q^2}{1+\alpha q} \int_0^a \xi P(r, \xi)\, d\xi + Q_0 P(r, a) \tag{1.17}$$

Here

$$P(r, \xi) = \int_0^\infty (\alpha^2 + 2q^2)^{-1/2} J_0(\alpha r) \sin \alpha\xi\, d\alpha \tag{1.18}$$

For the computation of the last integral we consider the expression

$$\frac{dP(r,\xi)}{d\xi} = \int_0^\infty (\alpha^2 + 2q^2)^{-1/2} \alpha J_0(\alpha r) \cos \alpha\xi\, d\alpha =$$

$$= -\int_0^\infty [1 - \alpha(\alpha^2 + 2q^2)^{-1/2}]\, J_0(\alpha r) \cos \alpha\xi d\alpha + \lambda (r^2 - \xi^2)^{-1/2} \tag{1.19}$$

$$\lambda = \begin{cases} 1, & \xi < r \\ 0, & \xi > r \end{cases}$$

Introducing the approximation (1.12) for $A_0 = b^2 = 1/2$ into the first term of (1.19) we have

$$\frac{dP(r,\xi)}{d\xi} \approx -q^2 \int_0^\infty (\alpha^2 + q^2)^{-1} J_0(\alpha r) \cos \alpha\xi\, d\alpha + \lambda (r^2 - \xi^2)^{-1/2} =$$

$$= -q \frac{\pi}{2} e^{-\xi q} I_0(rq) - \lambda q \Big[\operatorname{sh}(\xi q) \int_0^\beta \operatorname{ch}(rq \cos\varphi)\, d\varphi +$$

$$+ \operatorname{ch}(\xi q) \int_0^\beta \operatorname{sh}(rq\cos\varphi)\, d\rho - \frac{1}{q}(r^2 - \xi^2)^{-1/2} \Big] \tag{1.20}$$

$$(\beta = \arccos \xi / r)$$

Integrating the last equality with respect to ξ and taking into account that $P(r, 0) = 0$, we obtain

$$P(r,\xi) \approx \frac{\pi}{2} J_0(rq) e^{-\xi q} -$$

$$-\lambda \Big[\operatorname{ch}(\xi q) \int_0^\beta \operatorname{ch}(rq\cos\varphi)\, d\varphi - \operatorname{sh}(\xi q) \int_0^\beta \operatorname{sh}(rq\cos\varphi)\, d\varphi \Big] \tag{1.21}$$

Substituting (1.21) into (1.17) and performing some transformations, we find

$$T^*(r, 0) \approx \frac{Q_0}{1+aq} \Big[\frac{\pi}{2} \lambda_1 + (1-\lambda_1)\gamma - (1-\lambda_1)\, aq e^{aq} \int_0^{\beta_1} e^{-rq\cos\varphi} d\varphi \Big] \tag{1.22}$$

$$\gamma = \arcsin \frac{a}{r}, \qquad \beta_1 = \arccos \frac{a}{r}, \qquad \lambda_1 = \begin{cases} 1, & r < a \\ 0, & r > a \end{cases}$$

From the equality (1.22) it follows that

$$Q_0 = \frac{2}{\pi}(1 + aq)\, T_0^*(p) \tag{1.23}$$

Thus, the resultant expression for $T^*(r, 0, p)$ has the form

$$T^*(r,0,p)=\frac{2}{\pi}T_0^*(p)\left[\frac{\pi}{2}\lambda_1+(1-\lambda_1)\gamma-(1-\lambda_1)aqe^{aq}\int_0^{\beta_1}e^{-rq\cos\varphi}d\varphi\right] \tag{1.24}$$

If $T_0^*(p) = T_0 = \text{const}$, then, returning to the original in (1.24), we obtain

$$T(r,0,t)=\frac{2}{\pi}T_0\left\{\frac{\pi}{2}\lambda_1+(1-\lambda_1)\gamma-\right.$$

$$\left.-\frac{(1-\lambda_1)}{\sqrt{\pi}t^*}\int_0^{\beta_1}\exp\left[-\frac{1}{(2t^*)^2}\left(\frac{r}{a}\cos\varphi-1\right)^2\right]d\varphi\right\} \tag{1.25}$$

$$(t^*=a^{-1}\sqrt{2\varkappa t})$$

2. We proceed now to the determination of the state of stress of the unbounded space, situated under the action of the temperature field (1.25). It is known [2], that in the quasistationary case the elastic potential Φ satisfies the equation

$$\nabla^2\Phi=mT,\quad \nabla^2=\frac{\partial^2}{\partial r^2}+\frac{1}{r}\frac{\partial}{\partial r}+\frac{\partial^2}{\partial z^2} \tag{2.1}$$

Here

$$m=\frac{1+\nu}{1-\nu}\alpha_T$$

and α_T is the coefficient of linear expansion.

The displacements and the stresses in the space are determined by the formulas

$$u^\circ=\partial\Phi/\partial r,\qquad w^\circ=\partial\Phi/\partial z \tag{2.2}$$

$$\sigma_z^\circ=-2G\left(\frac{\partial^2\Phi}{\partial r^2}+\frac{1}{r}\frac{\partial\Phi}{\partial r}\right),\qquad \tau_{rz}^\circ=2G\frac{\partial^2\Phi}{\partial r\partial z} \tag{2.3}$$

From the symmetry of the state of stress it follows that $\tau_{rz}=0$ for $z=0$, and therefore the solution of Eq. (2.1) must satisfy the condition

$$\left.\frac{\partial\Phi}{\partial z}\right|_{z=0}=0 \tag{2.4}$$

Taking into account (1.5) and (2.4), we find

$$\Phi_0(r,z,p)=-\frac{\varkappa m}{p}\int_0^\infty B(\alpha)\left[\alpha^{-1}e^{-\alpha z}-(\alpha^2+2q^2)^{-1/2}e^{-z(\alpha^2+2q^2)^{-1/2}}\right]J_0(\alpha r)\,d\alpha \tag{2.5}$$

Making use of the expression (2.5), we determine the normal stresses $\sigma_z^{(0)}$ in the plane $z=0$

$$\sigma_z^{(0)}(r,0,p)=-2Gm\frac{\varkappa}{p}\int_0^\infty \alpha B(\alpha)\left[1-\alpha(\alpha^2+2q^2)^{-1/2}\right]J_0(\alpha r)\,d\alpha \tag{2.6}$$

Making use of the approxiamtion (1.12), we transform (2.6) into the form

$$\sigma_z^{(0)}(r,0,p)\approx-Gm\int_0^\infty\alpha(\alpha^2+q^2)^{-1}B(\alpha)J_0(\alpha r)\,d\alpha=\frac{2}{\pi}T_0^*(p)\,Gm\times$$

$$\times\left[q^2\int_0^a\xi\frac{dS(r,\xi)}{d\xi}d\xi+(1+aq)\left.\frac{dS(r,\xi)}{d\xi}\right|_{\xi=a}\right] \tag{2.7}$$

Here we have denoted

$$S(r,\xi)=\int_0^\infty(\alpha^2+q^2)^{-1}J_0(\alpha r)\cos\alpha\xi d\alpha \tag{2.8}$$

One can prove that

$$\frac{dS(r,\xi)}{d\xi}=-\frac{\pi}{2}e^{-\xi q}J_0(rq)+$$
$$+\lambda\left[\operatorname{ch}(\xi q)\int_0^\beta \operatorname{ch}(rq\cos\varphi)\,d\varphi-\operatorname{sh}(\xi q)\int_0^\beta \operatorname{sh}(rq\cos\varphi)\,d\varphi\right] \tag{2.9}$$

Substituting (2.9) into (2.7) and performing the necessary transformations, we obtain the final expression for the normal stress $\sigma_z^{(0)}(r, 0, p)$

$$\sigma_z^{(0)}(r,0,p)=-\frac{2}{\pi}T_0^*(p)Gm\times$$
$$\times\left[\frac{\pi}{2}\lambda_1+(1-\lambda_1)\gamma-(1-\lambda_1)aqe^{aq}\int_{a/r}^1(1-x^2)^{-1/2}e^{-rqx}dx\right] \tag{2.10}$$

3. We consider in the plane $z = 0$ of the unbounded space a circular slit of radius r_0 $(r_0 \geqslant a)$. We construct the axially symmetric state of stress, satisfying the following conditions:

$$\tau_{rz}^{(1)}=0,\qquad z=0 \tag{3.1}$$

$$\sigma_z^{(1)}=-\sigma_z^{(0)},\quad z=0,\quad r\leqslant r_0;\quad w^{(1)}=0,\quad z=0,\quad r>r_0 \tag{3.2}$$

We determine the components of the given state of stress in terms of the Love function with the formulas [6]

$$\begin{aligned}\tau_{rz}^{(1)}&=\frac{2G}{1-2\nu}\frac{\partial}{\partial r}\left[(1-\nu)\nabla^2-\frac{\partial^2}{\partial z^2}\right]\varphi_1\\ \sigma_z^{(1)}&=\frac{2G}{1-2\nu}\frac{\partial}{\partial z}\left[(2-\nu)\nabla^2-\frac{\partial^2}{\partial z^2}\right]\varphi_1\\ w^{(1)}&=\frac{1}{1-2\nu}\left[2(1-\nu)\nabla^2-\frac{\partial^2}{\partial z^2}\right]\varphi_1\end{aligned} \tag{3.3}$$

Here φ_1 is a biharmonic function which can be represented in the form

$$\varphi_1=\int_0^\infty[C(\alpha)+D(\alpha)\alpha z]\,e^{-\alpha z}J_0(\alpha r)\,d\alpha \tag{3.4}$$

Making use of the condition (3.1), we obtain

$$C(\alpha)=2\nu D(\alpha) \tag{3.5}$$

Imposing the conditions (3.2), we arrive at the dual equations

$$\int_0^\infty\alpha D_1(\alpha)J_0(\alpha r)\,d\alpha=-\frac{1-2\nu}{4G}\sigma_z^{(0)}(r,0,p),\qquad r\leqslant r_0 \tag{3.6}$$
$$\int_0^\infty D_1(\alpha)J_0(\alpha r)\,d\alpha=0,\qquad r>r_0\qquad(D_1(\alpha)=\alpha^2D(\alpha))$$

The solution of the dual equations (3.6) is known [7]. Therefore we bring the final result in the determination of the normal stresses $\sigma_z^{(1)}$ in the plane $z = 0$

$$\sigma_z^{(1)}(r,0,p) = \frac{4G}{1-2\nu}\frac{1}{r}\frac{d}{dr}\int_0^{\delta}\frac{\xi\psi(\xi)\,d\xi}{\sqrt{r^2-\xi^2}}, \qquad \delta = \begin{cases} r, & r<r_0 \\ r_0, & r>r_0 \end{cases} \tag{3.7}$$

Here

$$\psi(\xi) = -\frac{2}{\pi}\frac{1-2\nu}{4G}\int_0^{\xi}\frac{r\sigma_z^{(0)}(r,0,p)}{\sqrt{\xi^2-r^2}}\,dr \tag{3.8}$$

For the stresses $\sigma_z^{(1)}$, which appears outside the circle $r = r_0$,we obtain, according to (3.7), the expression

$$\sigma_z^{(1)}(r,0,p) = -\frac{4G}{1-2\nu}\frac{\psi(r_0)}{\sqrt{r^2-r_0^2}} + \frac{4G}{1-2\nu}\int_0^{r_0}\frac{\psi'(\xi)\,d\xi}{\sqrt{r^2-\xi^2}}, \qquad r>r_0>a \tag{3.9}$$

Substituting (2.10) into the formula (3.8), we find

$$\psi(\xi) = -\left(\frac{2}{\pi}\right)^2\frac{1-2\nu}{4}T_0^*(p)\left\{-\frac{\pi}{2}\xi + \lambda^\circ\left[a(1+aq)\int_a^{\xi}\sqrt{\frac{\xi^2-\eta^2}{\eta^2-a^2}}\frac{d\eta}{\eta} - \right.\right.$$

$$\left.\left. - aq^2e^q\int_0^{\xi}\sqrt{\xi^2-a^2}\int_0^{\zeta}e^{-\eta q\cos\varphi}\cos\varphi\,d\varphi\,d\eta\right]\right\}, \qquad \lambda^\circ = \begin{cases} 1, & \xi>a \\ 0, & \xi<a \end{cases} \tag{3.10}$$

$$(\zeta = \arccos a/\eta)$$

Obviously, the solution of the formulated problem can be represented in the form of the sum of the two states of stress considered above, so that in the limiting case the stress intensity factor at the top of the disk-shaped slit is

$$K_{IC} = \lim_{r\to r_0}\sqrt{2\pi(r-r_0)}\,[\sigma_z^{(1)}(r,0,p) + \sigma_z^{(0)}(r,0,p)] = \left(\frac{2}{\pi}\right)^2\sqrt{\frac{\pi}{r_0}}\,T_0^*(p)Gm\times$$

$$\times\left[-\frac{\pi}{2}r_0 + a\int_a^{r_0}\sqrt{\frac{r_0^2-\eta^2}{\eta^2-a^2}}\frac{d\eta}{\eta} + aqe^{aq}\int_0^{r_0}\frac{\eta\,d\eta}{\sqrt{r_0^2-\eta^2}}\int_0^{\zeta}e^{-\eta q\cos\varphi}d\varphi\right] \tag{3.11}$$

If $T_0^*(p) = T_0 = \text{const}$,then, returning in (3.11) to the original,we obtain

$$\frac{1}{\bar{T}} = \frac{1}{\sqrt{\lambda}}\left[-\frac{\pi}{2} + \int_1^{\lambda}\sqrt{\frac{\lambda^2-x^2}{x^2-1}}\frac{dx}{x} + \right.$$

$$\left. + \frac{1}{t^*\sqrt{\pi}}\int_1^{\lambda}\frac{x\,dx}{\sqrt{\lambda^2-x^2}}\int_0^{\varepsilon}\exp\left[-\frac{(x\cos\varphi-1)^2}{4(t^*)^2}\right]d\varphi\right] \tag{3.12}$$

$$\lambda = \frac{r_0}{a}, \qquad \bar{T} = 4\pi^{-3/2}a^{1/2}mT_0K_{IC}^{-1}G, \qquad \varepsilon = \arccos\frac{1}{x}$$

In Fig. 1 we have represented in dimensionless coordinates, according to (3.12), the length of the crack as a function of temperature (corresponding to the loads in these problems) at different times. It turns out that for every value t^* there exists some critical value $\lambda = \lambda_i^*$ of the ratio between the length of the crack and the length of the thermal "filling", for which the infinite stresses are absent at the end of the crack (the vertical asymptote corresponds to the case $K_I = 0$). Only in the case $\lambda > \lambda_i^*$ does there occur a growth of the crack, having a Griffith character. We note that for $t^* > 0$ the curves of this dependence tend sufficiently fast to the common asymptotic line ($t^* = 0, \infty$), practically running together with it already for $t^* = 10$. For $\lambda \to \infty$ we have $a \to 0$, which corresponds to the instantaneous pointwise application of the temperature $T \to 2\pi^{-1}\lambda^{-1/2}$. For $\lambda < \lambda_i^*$ compression stresses arise and the growth of the crack does not take place. In Fig. 3 we have represented the limiting length of the crack as function of time for $\bar{T}$ = const . For each specific value of the temperature, the specification of the initial crack length $\lambda_0 < \lambda_i^0$ will insure prevention of fracture for an arbitrary time range of service of parts with a crack under the conditions of such loading.

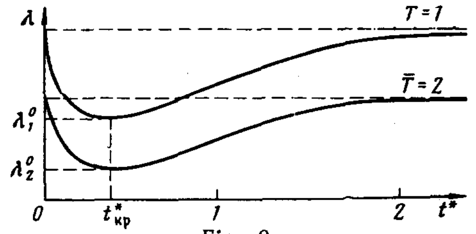

Fig. 3.

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